An Alternative Approach to the Search for Stochastic Signals

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Why "alternative"?

- Stochastic background is a random signal, characterised by the gravitational wave spectrum $\Omega_{gw}(f) = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df}$
- In one detector, stochastic signal indistinguishable from detector noise
- Standard search uses the cross-correlation statistic

$$\tilde{s}_I(f_K; t_J) = \tilde{n}_I(f_K; t_J) + \tilde{h}_I(f_K; t_J) \qquad Y = \int \tilde{s}_1^*(f)\tilde{s}_2(f)\tilde{Q}(f)df \propto \Omega$$

- $\tilde{Q}(f)$ is the *optimal filter function* and combines
  - Instrumental noise PSDs $P_{1,2}(f)$
  - Overlap reduction function $\gamma(f)$
  - Shape of spectrum – matched filter search

The alternative

- We apply Bayes' theorem directly to the IFO data – no filter function and no statistic constructed

Analysis approach

Likelihood

$$p(\vec{d}|\vec{\theta}) = \prod_J \prod_K (2\pi \det \mathbf{C}_{JK})^{-N/2} \exp\left(-\frac{1}{2}\mathbf{s}_{JK}^T \mathbf{C}_{JK}^{-1} \mathbf{s}_{JK}\right)$$

$$\mathbf{C}_{JK} = \begin{pmatrix} \sigma_1^2(f_K;t_J) + \sigma_h^2(f_K) & \gamma_{12}(f_K)\sigma_h^2(f_K) & \dots & \gamma_{1N}(f_K)\sigma_h^2(f_K) \\ \gamma_{21}(f_K)\sigma_h^2(f_K) & \sigma_2^2(f_K;t_J) + \sigma_h^2(f_K) & \dots & \gamma_{2N}(f_K)\sigma_h^2(f_K) \\ \vdots & \vdots & \ddots & \\ \gamma_{N1}(f_K)\sigma_h^2(f_K) & & & \sigma_N^2(f_K;t_J) + \sigma_h^2(f_K) \end{pmatrix},$$

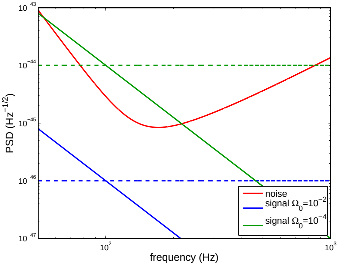
- $\mathbf{s}_{JK} = (\tilde{s}_1(f_K;t_J), \tilde{s}_2(f_K;t_J), \dots \tilde{s}_N(f_K;t_J))^T$
- $\tilde{s}_I(f_K;t_J) = \tilde{n}_I(f_K;t_J) + \tilde{h}_I(f_K;t_J)$
- $\sigma_I^2 \propto P_I(f)$
- $\sigma_h^2 \propto \frac{\Omega_{gw}(f)}{f^3}$

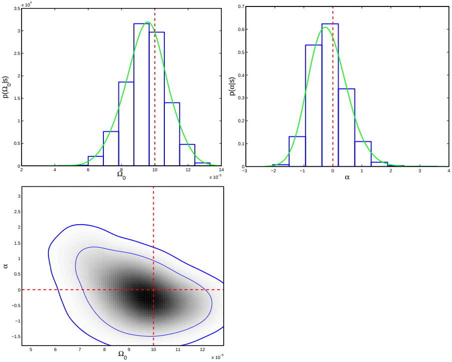
Toy model

- Simulated data with Gaussian stationary coloured noise (using LIGO SRD) and signal (assuming colocated, coincident detectors)
- Signal model is a power law

$$\Omega_{gw}(f) = \Omega_0 \left(\frac{f}{f_0} \right)^{\alpha}$$

- Explore PDFs with MCMCs
- Assume noise known





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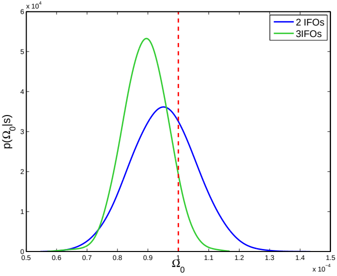

- Multiple detectors
- Increase from 2 to 3 IFOs reduces width of posterior by $\approx \sqrt{2}$
- Simple to include correlated noise (although noise on the level of H1-H2 would probably dominate over signal)

Next steps...

Unknown noise:

- Parameterise noise and estimate? – non-stationarity of noise leads to huge numbers of parameters
- Estimate from adjacent segments (as in CC analysis)?
  - This is more promising – large signal tests have shown that this approach does indeed work
  - However, in the small-signal limit, the error in the estimate of the PSD dominates. We need several hundreds of FFTs for the estimation of the noise to be sufficiently accurate for this analysis.

Extensions:

- Correlated detector noise
- Anisotropic search

END