Random template banks and relaxed lattices
or: why it's efficient to be lazy

C. Messenger, R. Prix, M. A. Papa

Albert-Einstein-Institut

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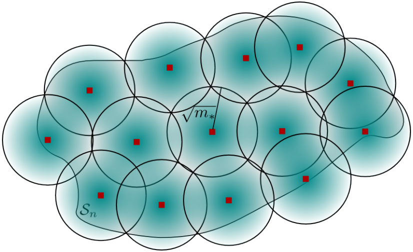
Matched filtering with templates $h(t;\lambda)$



Signal in $\lambda_s \in \mathcal{S}_n$
targeting $\lambda = \lambda_s + \Delta\lambda$

☞ loss: $\mathrm{SNR}(\lambda) < \mathrm{SNR}(\lambda_s)$

mismatch $m \equiv 1 - \frac{\mathrm{SNR}(\lambda)}{\mathrm{SNR}(\lambda_s)}$

if offset $\Delta\lambda$ "small" $\Longrightarrow$ $m = g_{ij}\,\Delta\lambda^i\,\Delta\lambda^j \equiv |\Delta\lambda|^2$

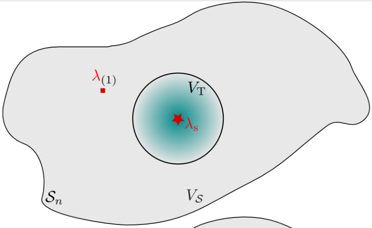
Template bank of given mismatch m_*

Each template $\lambda_{(k)}$ *covers* a sphere $|\lambda - \lambda_{(k)}|^2 \le m_*$

Template bank(m_*):= cover $\mathcal{S}_n$ with spheres of radius $\sqrt{m_*}$

Number of templates: $N = \theta\, m_*^{-n/2}\, V_{\mathcal{S}}$
(normalized thickness θ of a covering)

Random template bank construction

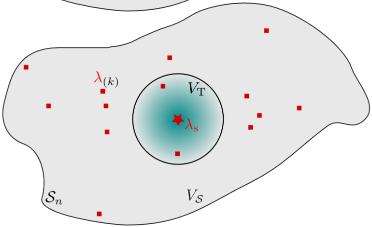


Place one template randomly

$$P(\,\text{miss}\,|\,N_{\mathcal{R}} = 1) = 1 - \frac{V_T}{V_S}$$

$$P(\,\text{hit}\,|\,N_{\mathcal{R}} = 1) = \frac{V_T}{V_S}$$

(covered volume $V_T = m_*^{n/2}\, V_n$)



Place $N_{\mathcal{R}}$ template randomly

$$P(\,\text{miss}\,|\,N_{\mathcal{R}}) = \left(1 - \frac{V_T}{V_S}\right)^{N_{\mathcal{R}}}$$

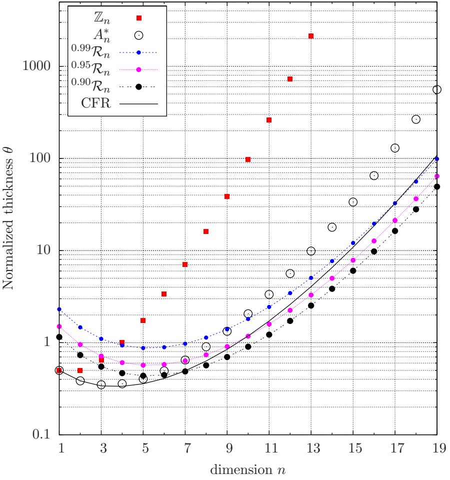
$$P(\,\text{hit}\,|\,N_{\mathcal{R}}) = 1 - \left(1 - \frac{V_T}{V_S}\right)^{N_{\mathcal{R}}}$$

Choose a "covering confidence" $\eta = P(\,\text{hit}\,|\,N_{\mathcal{R}}) < 1$

☞ determines $N_{\mathcal{R}}(\eta, m_*) \approx \underbrace{\frac{1}{V_n} \ln\left(\frac{1}{1-\eta}\right)}_{\text{thickness } \theta(\eta)} m_*^{-n/2}\, V_S$

Properties of random template banks

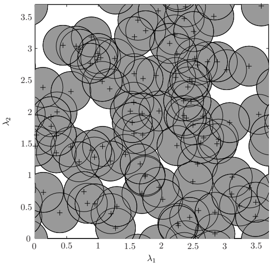
${}^{\eta}\mathcal{R}_n(m_*)$: Random-template bank for mismatch m_* and covering confidence η: thickness $\theta(\eta) = \frac{1}{V_n} \ln\left(\frac{1}{1-\eta}\right)$



- ${}^{\eta}\mathcal{R}_n$ beats best lattice covering at higher dimensions
- beats theoretical lower limit (CFR) on covering thickness
- possible because not strictly a covering
- major practical advantage: simplicity
- if non-constant metric $g_{ij}(\lambda)$: sampling density $\propto \sqrt{\det g_{ij}}$

Spatial coverage and worst-case mismatch

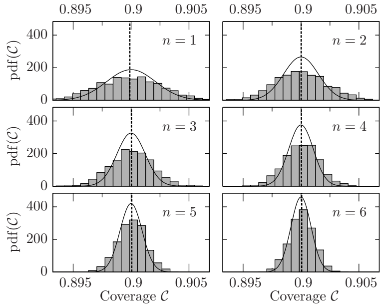
example: ${}^{0.9}\mathcal{R}_n(0.1)$

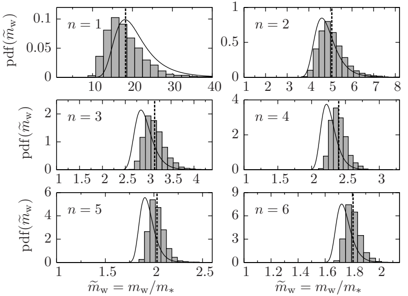


${}^{\eta}\mathcal{R}_n$ provide incomplete coverage
Statistics of individual realizations:

- Spatial fraction $\mathcal{C}$ of parameter-space $\mathcal{S}_n$ covered. Can show $E[\mathcal{C}] = \eta$.
- worst-case mismatch "hole" $m_{\rm w}$

${}^{0.9}\mathcal{R}_n$ Monte-Carlo simulations for $N_{\mathcal{R}} = 10^4$

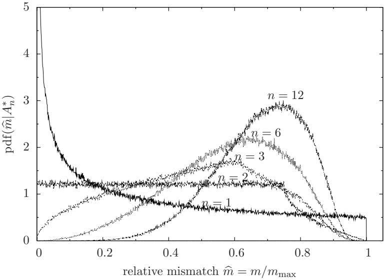


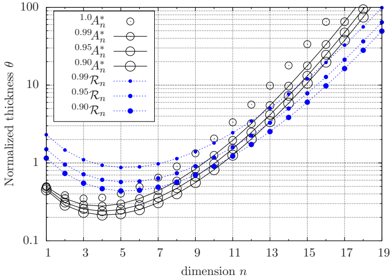


Lesson from ${}^{\eta}\mathcal{R}_n$: efficient to be "lazy". Covering the "last few percent" of $\mathcal{S}_n$ gets increasingly costly at higher dimensions

☞ give up complete coverage for lattices

Use MC-simulation to find $\mathrm{pdf}(m|\Lambda_n)$: find "relaxed" mismatch m_* such that $P(m \leq m_*) = \eta \implies$ allows to use coarser lattice





☞ Very efficient, but only at lower dimensions ($n \lesssim 10$)

☞ more difficult than ${}^{\eta}\mathcal{R}_n$, especially with varying metric $g_{ij}(\lambda)$