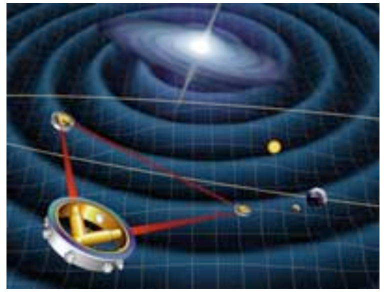
Introduction



Source for LISA : $10^{-4} - 10^{-2}$ Hz

Capture of :

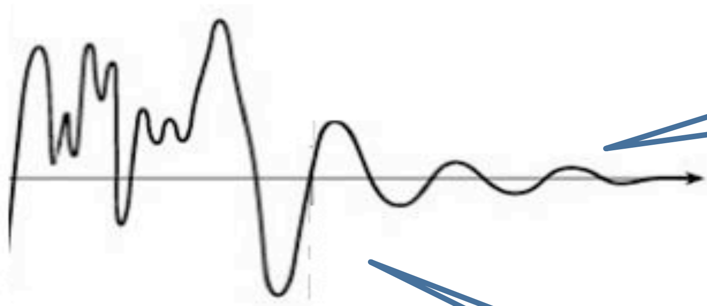
$7\ 10^{-3}$ Hz < f < $3\ 10^{-2}$ Hz

- white dwarfs $\mu = 0.6\ M_{\odot}$
- neutron stars $\mu = 1.4\ M_{\odot}$
- black holes $\mu = 10 - 100\ M_{\odot}$

SMBH $M = 10^5 - 10^7\ M_{\odot}$

Orbital parameters

The univers geometry



rotation

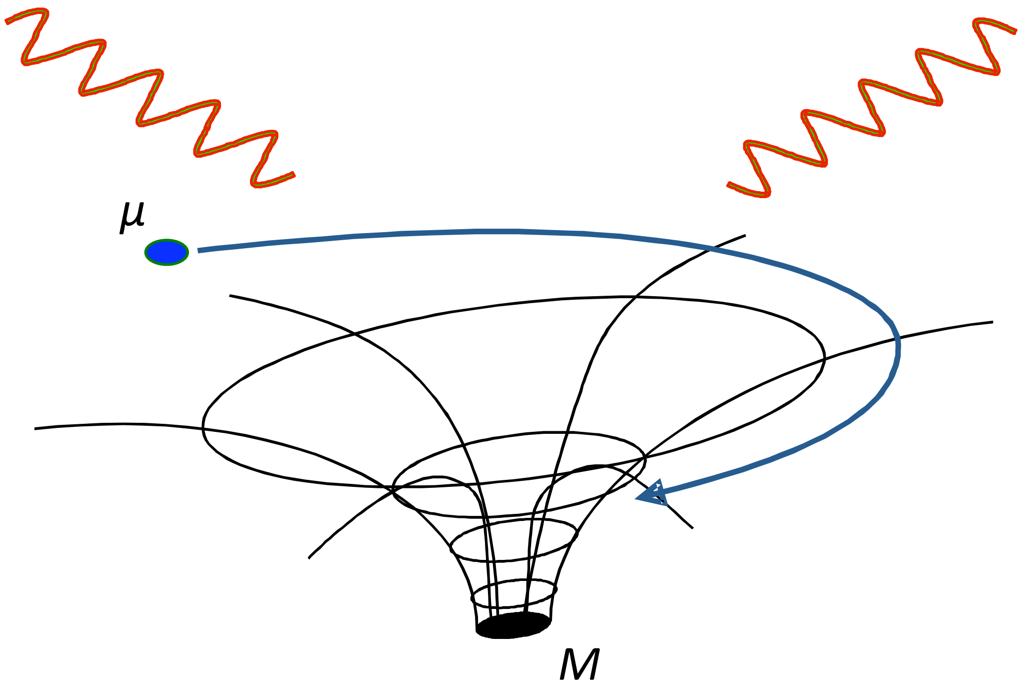
relaxation

plunge

Horizon

Introduction

To extract all informations it is supposed to know accurately the orbital evolution of the system:



Include the effect of

radiation reaction or **self-force**

$$\mu \ll M$$

Low mass ratio ⇒ use of
- linear perturbations
- pointlike partice

Linearization

Let's consider linear perturbations induced by a pointlike particle of mass μ

$$\mathbf{g}_{\alpha\beta} = g_{\alpha\beta} + h_{\alpha\beta} \qquad h_{\mu\nu} \ll g_{\mu\nu}$$

$$\delta T^{\mu\nu} = \mu \frac{dT}{d\tau}\frac{dz^\mu}{dt}\frac{dz^\nu}{dt}\frac{\delta\left[r - R(t)\right]}{r^2}\delta\left[cos\theta - cos\Theta(t)\right]\,\delta\left[\phi - \Phi(t)\right]$$

- spherical harmonics expansion
- decoupling on even parity $H_0,\ H_1,\ H_2,\ K,\ G,\ h_0$ et h_1 and odd parity $h_0^o,\ h_1^o$ et h_2^o
- choice of the Regge-Wheeler gauge : $h_0 = h_1 = h_2^o = G = 0$

$$\mathbf{R}_{\mu\nu} - \frac{1}{2}\mathbf{g}_{\mu\nu}\mathbf{R} = -8\pi\mathbf{T}_{\mu\nu}$$

system of 10 equations : 7 of even parity and 3 of odd parity

$$\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial r_*^2} + V^e(r)\psi = S(t,r)$$

Zerilli equation

$$\frac{\partial^2 R}{\partial t^2} - \frac{\partial^2 R}{\partial r_*^2} + V^o(r)R = S^o(t,r)$$

Regge-Wheeler-(Zerilli) equation

Pointlike particle approximation

Even if the concept of pointlike particle avoids describing the internal structure of the captured object, it gives rise to some difficulties.

The perturbations metric diverges at the particle position

$$h_{\alpha\beta} = \frac{\mu}{R}$$

(R : distance between the particle and a point of the fielf)

- The linear perturbations approximation becomes invalid.
- This divergence becomes a divergence of the sum over the modal components.
- Impossibility to défine the motion.

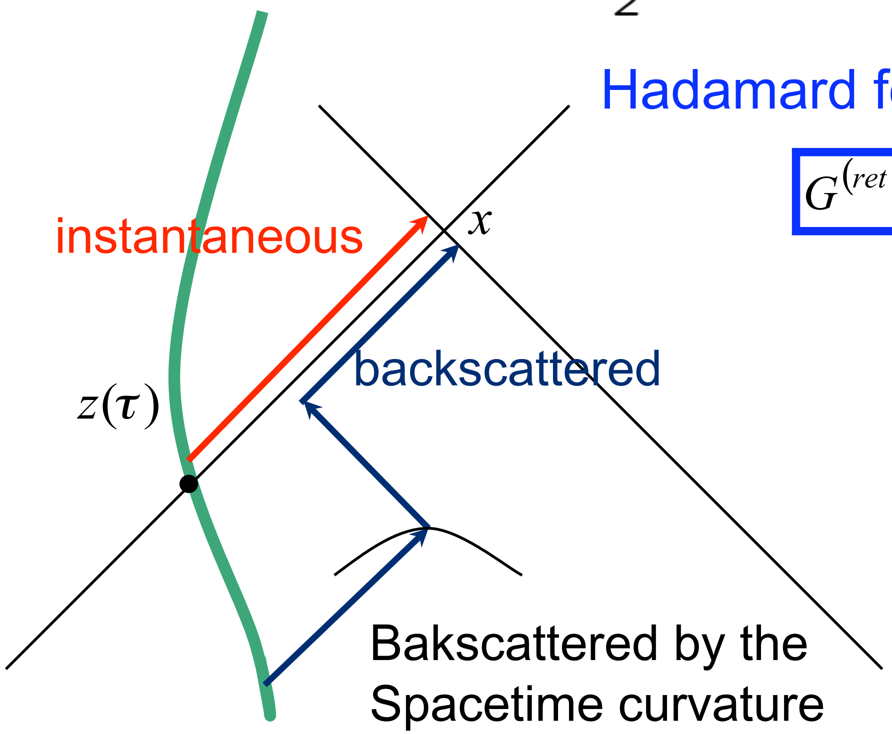
This problem has been solved by the matched asymptotic expansion, the axiomatic approach or by the method of the radiation field.

Backscattered perturbations

The equation of motion of a pointlike particle is given by :

$$\frac{Du^\mu}{d\tau} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(2h^R_{\nu\lambda\,;\rho} - h^R_{\lambda\rho\,;\nu}\right)u^\lambda u^\rho$$

$$= -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(2h^{diff}_{\nu\lambda\,;\rho} - h^{diff}_{\lambda\rho\,;\nu}\right)u^\lambda u^\rho$$



Hadamard form of the retarded Green functions

$$G^{(ret)\mu\nu\alpha\beta}(x,z) = U^{\mu\nu\alpha\beta}\delta[\sigma(x,z)] + V^{\mu\nu\alpha\beta}\theta[\sigma(x,z)]$$

$$h^{\mu\nu}_{diff}(x) \approx \int_{-\infty}^{\tau-} d\tau' V^{\mu\nu}{}_{\alpha\beta}[x, z(\tau')]\, T^{\alpha\beta}[z(\tau')]$$

$$\tau^- = \tau - \epsilon$$

Régularization (*mode sum*)

The retarded and instantaneous solutions have the same singular behavior near the world line

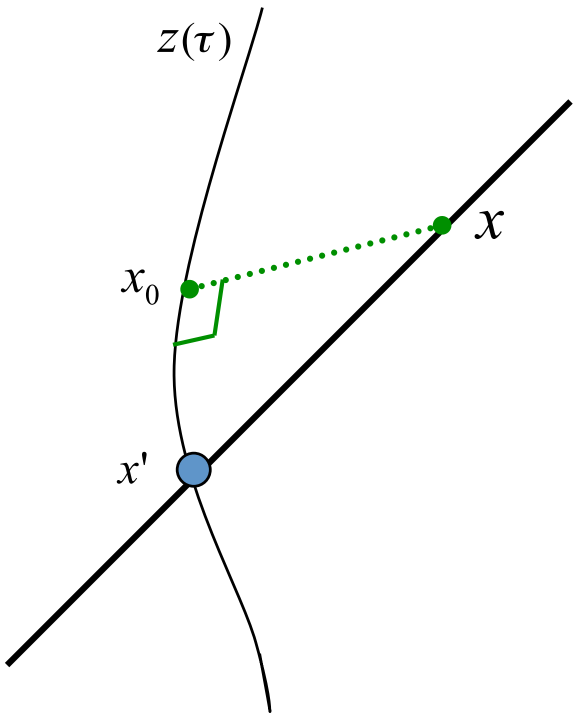
$$F_\alpha = F_\alpha^{tot} - F_\alpha^{inst} = \mu^2 \int_{-\infty}^{\tau^-} k_\alpha{}^{\beta\gamma\delta}\, G^+{}_{\beta\gamma\beta'\gamma';\delta}(x;x')\, u^{\beta'} u^{\gamma'} d\tau'$$

$$F_\alpha = \lim_{\epsilon\to 0^+} \sum_{l=0}^{\infty} \left(F_\alpha^{tot\ l} - \delta F_\alpha^{(\epsilon)\ l} \right)$$

converges

$$= \sum_{l=0}^{\infty} \left(F_\alpha^{tot\ l} - H_\alpha^l \right) + D_\alpha$$

$$D_\alpha = \lim_{\epsilon\to 0^+} \sum_{l=0}^{\infty} \left(H_\alpha{}^l - \delta F_\alpha^{(\epsilon)\ l} \right)$$



The function *H* has the same asymptotic behavior as the modal components of the instantaneous self-force near the particle position and for large values of l

Geodesic deviation

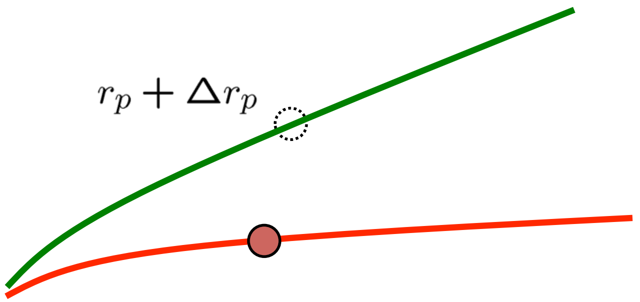
Background motion + self-force = geodesic motion with respect to the total metric

$$\mathbf{g}_{\alpha\beta} = g_{\alpha\beta} + h^R_{\alpha\beta}$$

Geodesic approch :

$$r = r_p + \Delta r_p \qquad \mathbf{g}_{\mu\nu}(r) = \mathbf{g}_{\mu\nu}(r_p) + \Delta r_p (\partial \mathbf{g}_{\mu\nu}/\partial r)_{r_u}$$

$$\frac{d^2 r}{dt^2} = \Gamma^t_{rr}\left(\frac{dr}{dt}\right)^3 + \left(2\Gamma^t_{tr} - \Gamma^r_{rr}\right)\left(\frac{dr}{dt}\right)^2 + \left(\Gamma^t_{tt} - 2\Gamma^r_{tr}\right)\left(\frac{dr}{dt}\right) - \Gamma^r_{tt}$$



$$\Delta \ddot{r}_p = \alpha_1 [g, \dot{r}_p] \, \Delta r_p + \alpha_2 [g, \dot{r}_p] \, \Delta \dot{r}_p + \alpha_6 [h, \dot{r}_p]$$

$$\frac{Du^\mu}{d\tau} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(2h^R_{\nu\lambda\,;\rho} - h^R_{\lambda\rho\,;\nu}\right) u^\lambda u^\rho$$

$$\frac{d^2}{d\tau^2}\left(x^\alpha + \sigma^\alpha \Delta r\right) + \frac{1}{2}\left(g^{a\rho} + \Delta r g^{a\rho}_{,r}\right)\left[\left(g_{\beta\rho\,,\gamma} + \Delta r\, g_{\beta\rho\,,\gamma r}\right) + \left(g_{\gamma\rho\,,\beta} + \Delta r\, g_{\gamma\rho\,,\beta r}\right)\right.$$
$$\left. - \left(g_{\beta\gamma\,,\rho} + \Delta r\, g_{\beta\gamma\,,\rho r}\right)\right] \frac{d}{d\tau}\left(x^\beta + \sigma^\beta \Delta r\right) \frac{d}{d\tau}\left(x^\gamma + \sigma^\gamma \Delta r\right) \frac{d^2 x^\alpha}{d\tau^2}$$
$$+ {}^0\Gamma^\alpha_{\beta\gamma} u^\beta u^\gamma + \sigma^\alpha \frac{d^2 \Delta r}{d\tau^2} + \Delta r \Lambda^\alpha_{\beta\gamma} u^\beta u^\gamma + 2\frac{d\Delta r}{d\tau}\, {}^0\Gamma^\alpha_{\beta\gamma} u^\beta \sigma = -\frac{\mu}{2}\left(g^{\alpha\beta} + u^\alpha u^\beta\right)\left(2h_{\beta\gamma\,;\delta} - h_{\delta\gamma\,;\beta}\right) u^\gamma u^\delta$$

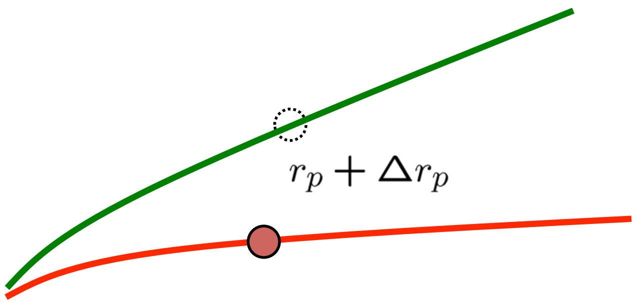
$$\sigma^\alpha = (0, 1, 0, 0)$$

Geodesic deviation

$$u^c\nabla_c(u^b\nabla_b Z^a) = \frac{1}{2M}R_{bcd}{}^a S^{bc}u^d - R_{bcd}{}^a u^b Z^c u^d - (g^{ab}+u^a u^b)(\nabla_d h^{\rm tail}_{bc} - \frac{1}{2}\nabla_b h^{\rm tail}_{cd})u^c u^d$$

$$\frac{d^2}{d\tau^2}(x^\alpha + \sigma^\alpha \Delta r) + \frac{1}{2}\left(g^{a\rho} + \Delta r g^{a\rho}_{,r}\right)\left[(g_{\beta\rho\,,\gamma} + \Delta r\, g_{\beta\rho\,,\gamma r}) + (g_{\gamma\rho\,,\beta} + \Delta r\, g_{\gamma\rho\,,\beta r})\right.$$
$$\left. - (g_{\beta\gamma\,,\rho} + \Delta r\, g_{\beta\gamma\,,\rho r})\right]\frac{d}{d\tau}\left(x^\beta + \sigma^\beta \Delta r\right)\frac{d}{d\tau}(x^\gamma + \sigma^\gamma \Delta r)\,\frac{d^2 x^\alpha}{d\tau^2}$$
$$+{}^0\Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma + \sigma^\alpha\frac{d^2\Delta r}{d\tau^2} + \Delta r \Lambda^\alpha_{\beta\gamma}u^\beta u^\gamma + 2\frac{d\Delta r}{d\tau}{}^0\Gamma^\alpha_{\beta\gamma}u^\beta\sigma = -\frac{\mu}{2}\left(g^{\alpha\beta} + u^\alpha u^\beta\right)\left(2h_{\beta\gamma\,;\delta} - h_{\delta\gamma\,;\beta}\right)u^\gamma u^\delta$$

$$\sigma^\alpha = (0,1,0,0)$$



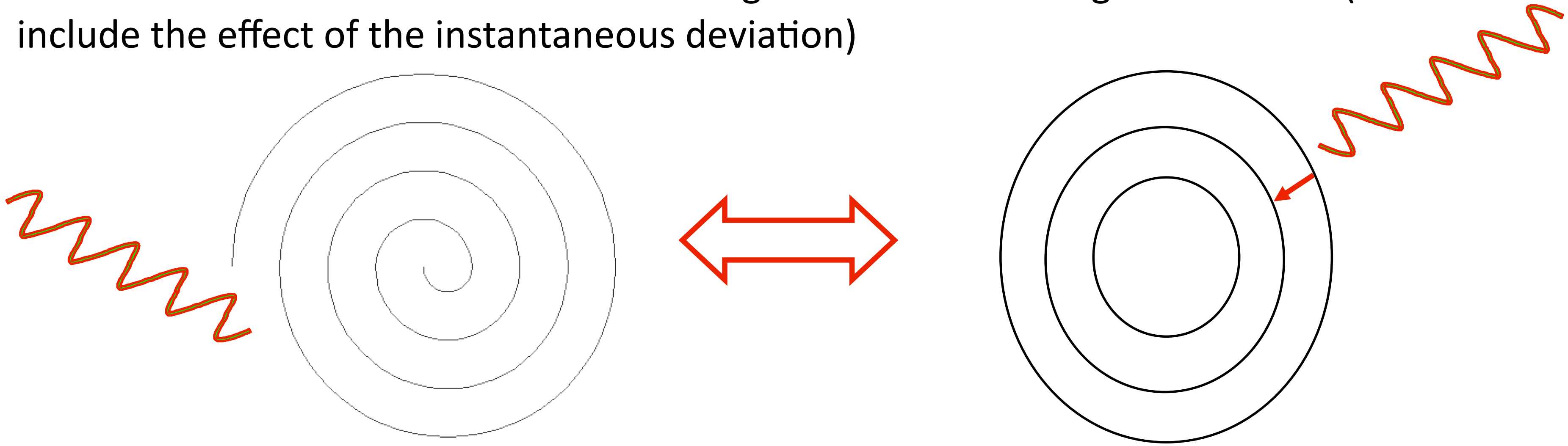
$$\frac{Du^\mu}{d\tau} = -\frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(2h^R_{\nu\lambda\,;\rho} - h^R_{\lambda\rho\,;\nu}\right)u^\lambda u^\rho$$

Geodesic deviation

Consider or not the geodesic deviation ?

Everything is a matter of time scale

• Circular orbit : time scale of the radiation reaction is greater than the orbital time scale. We can consider the orbital evolution as a geodesic of the background metric (does not include the effect of the instantaneous deviation)



• Radial infall: time scale of the radiation reaction lower than the infall time. It is necessar to consider the geodesic deviation during the infall (with some care).