Optimizing sensitivity of searches for continuous gravitational waves at fixed computing cost

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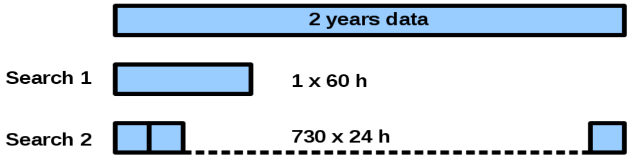
Wide parameter search for Continuous Gravitational Waves

- We are interested in detection of unknown sources of continuous gravitational waves, e.g. unknown pulsars.
- To perform a search, we need to compute a matched filter ($\mathcal{F}$-statistic).
- Even simple blind search for unknown pulsars is a search over very large 4D-parameter space

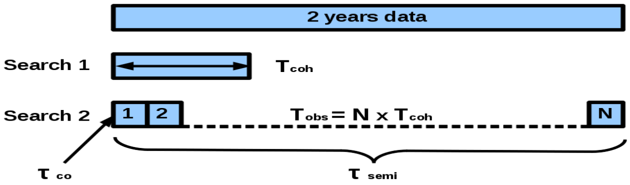
$$\{\alpha, \delta, f, \dot{f}\}.$$

- Full coherent integration is computationally limited, thus semicoherent techniques should be applied (e.g. Einstein@Home).



How much data should be used to maximize the sensitivity at fixed computing cost?

Search sensitivity at fixed computing cost



- Computing time of single segments

$$\tau_{coh} = \tau_c^0 T_{coh} \mathcal{N}(T_{coh}) \tag{1}$$

- Computing cost τ_{semi} to combine N segments

$$\tau_{semi} = N \tau_s^0 \gamma(N) \mathcal{N}(T_{coh}) \tag{2}$$

- Total computing cost

$$\tau(N, T_{coh}) = N\tau_{coh} + \tau_{semi} \tag{3}$$

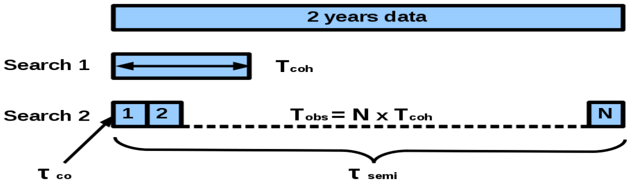
$\mathcal{N}(T_{coh})$ - number of templates

$\gamma(N)$ - fine grid refinement

τ_c^0 - time to operate on coarse grid point

τ_s^0 - time to operate on refined grid point

Search sensitivity at fixed computing cost



Assume

Number of coarse gird templates $\mathcal{N}(T_{coh}) \propto T^{\beta}$

Refinement factor $\gamma(N) \propto N^{s(s+1)/2}$, with s spindowns [BC,CGK,HP]

- Total computing cost

$$\tau(N, T_{coh}) = \kappa_c N T_{coh}^{\beta+1} + \kappa_s N^{\alpha} T_{coh}^{\beta} \tag{4}$$

- Sensitivity of a search

$$h_0(N, T_{coh}) = C\sqrt{\frac{S_n}{T_{coh} N^{1/2}}}, \tag{5}$$

where h_0 is the minimal measurable strain

Analytical solution

- If the coherent, resp. semicoherent part of the total computing cost dominates

$$\tau(N, T_{coh}) = \kappa N^{\eta} T_{coh}^{\delta} \tag{6}$$

- The number of segments as function of τ and T_{obs} is

$$N(\tau, T_{obs}) = \left(\frac{\tau}{\kappa}\right)^{1/(\eta-\delta)} T_{obs}^{\delta/(\delta-\eta)}. \tag{7}$$

- Substitution in the sensitivity equation yields

$$h_0(\tau, T_{obs}) \propto \tau^{-1/4(\delta-\eta)} T_{obs}^{-(\delta-2\eta)/4(\delta-\eta)}. \tag{8}$$

Analytical solution

- We could read out a critical condition

$$\delta > 2\eta \tag{9}$$

for which the optimal amount of data to use is all of the data taken by the instrument!

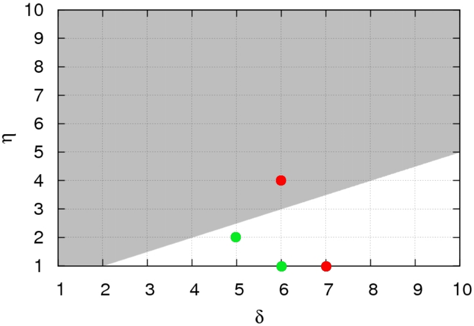


Figure: *Evaluation of the critical condition*

Symbol	Search	λ	Coherent (η, δ)	Semicoherent (η, δ)	Critical condition
● (green)	Einstein@Home	$\alpha, \delta, f, \dot{f}$	$(1, 6)$	$(2, 5)$	holds
● (red)	Cassiopeia A	$f, \dot{f}, \ddot{f}$	$(1, 7)$	$(4, 6)$	violated by $(4, 6)$

Analytical solution

- If the critical condition is violated, apply the method of Lagrange multipliers
- The Lagrange function is given by

$$\mathcal{L}(N, T_{coh}, \Lambda) = C\sqrt{\frac{S_n}{T_{coh}N^{1/2}}} + \Lambda(\tau(N, T_{coh}) - \tau_0) \tag{10}$$

- Solve

$$\partial_N \mathcal{L}(N, T_{coh}, \Lambda) = 0 \tag{11}$$

$$\partial_{T_{coh}} \mathcal{L}(N, T_{coh}, \Lambda) = 0 \tag{12}$$

$$\partial_\Lambda \mathcal{L}(N, T_{coh}, \Lambda) = 0 \tag{13}$$

to find N and T_{coh}

- The optimal amount of total observation data

$$T_{obs} = \frac{\beta - 2\alpha}{1-\beta}\frac{\kappa_s}{\kappa_c}\left[\frac{1-\beta}{1-2\alpha}\frac{\tau_0}{\kappa_s}\left(\frac{\beta-2\alpha}{1-\beta}\frac{\kappa_s}{\kappa_c}\right)^{-\beta}\right]^{\frac{\alpha}{\alpha+\alpha\beta-\beta}} \tag{14}$$

Example: Einstein@Home - search for unknown pulsars

- It is a search over $\{\alpha, \delta, f, \dot{f}\}$
- For $\tau(N, T_{coh}) = \kappa_c N T_{coh}^6$

$$h_0(\tau_0, T_{obs}) \propto \tau_0^{-1/20} T_{obs}^{-1/5}. \quad (15)$$

- For $\tau(N, T_{coh}) = \kappa_s N^2 T_{coh}^5$

$$h_0(\tau_0, T_{obs}) \propto \tau_0^{-1/12} T_{obs}^{-1/12}. \quad (16)$$

- h_0 is monotonically decreasing function of the observation time T_{obs}

Example: Einstein@Home - search for unknown pulsars

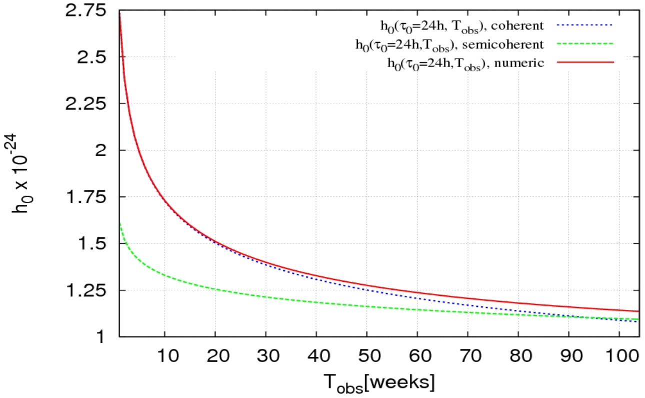


Figure: *Numerical solution Einstein@Home*

Use all the data!

Example: Cassiopeia A - a targeted search

- It is a search over $\{f, \dot{f}, \ddot{f}\}$
- For $\tau(N, T_{coh}) = \kappa_c N T_{coh}^7$

$$h_0(\tau_0, T_{obs}) \propto \tau_0^{-1/24} T_{obs}^{-5/24}. \tag{17}$$

- For $\tau(N, T_{coh}) = \kappa_s N^4 T_{coh}^6$

$$h_0(\tau_0, T_{obs}) \propto \tau_0^{-1/8} T_{obs}^{1/4}, \tag{18}$$

- For $\tau_0 = 24h$, h_0 is minimized by using $T_{obs} = 28.5$ weeks or 199.7 days of data

Example: Cassiopeia A - a targeted search

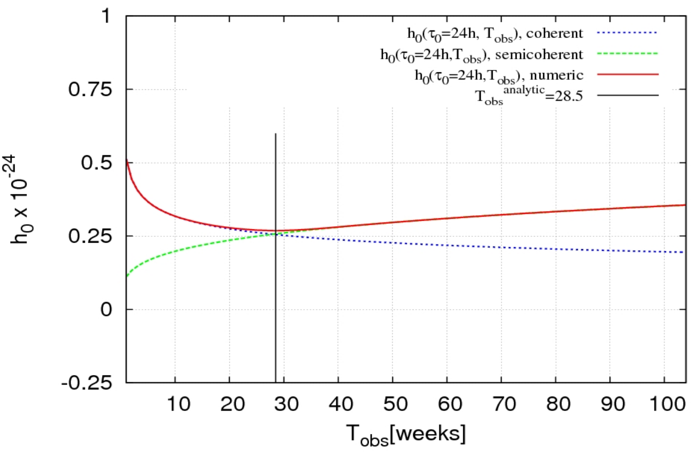


Figure: *Numerical solution Cassiopeia A*

Use 28.5 weeks of data!

Summary

- We derived a critical condition which defines class of searches for which the optimal amount of data at fixed computing cost is all of the available data.
- For all other cases, by using the method of Lagrange multipliers, we found analytical expression, which gives the optimal amount of data as function of the searched volume and total computing time.