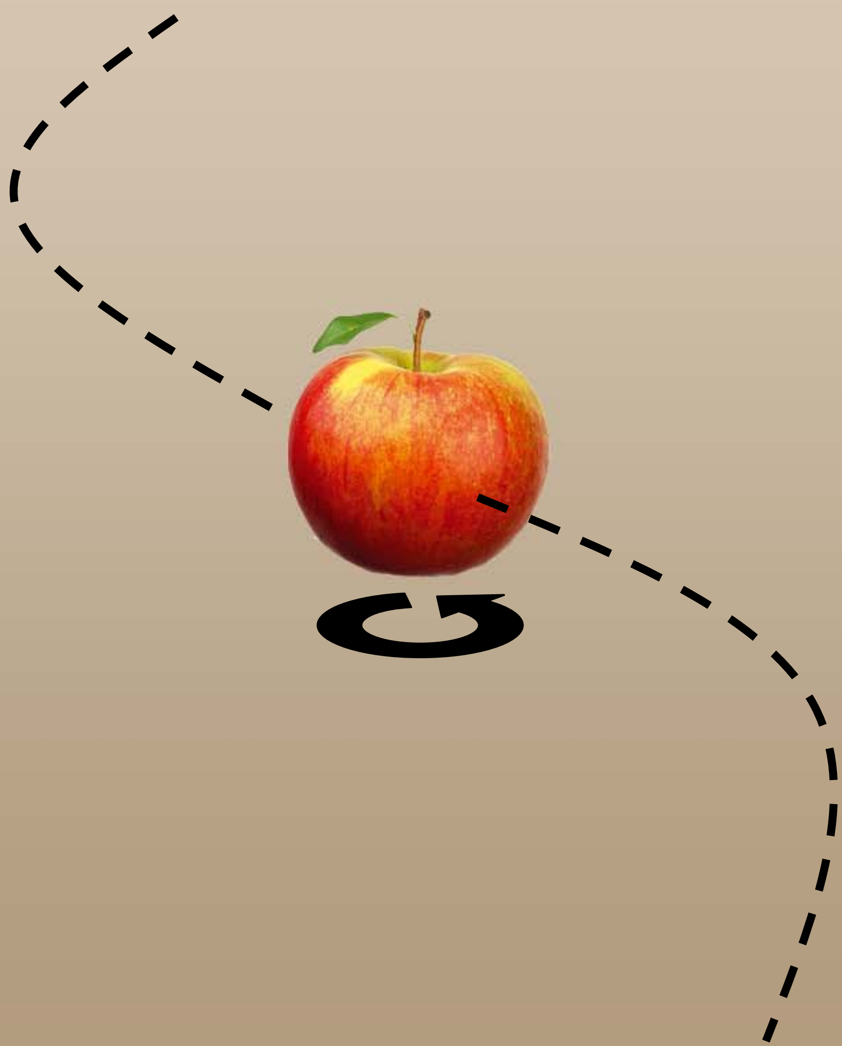

On the gravitational multipolar approximation technique

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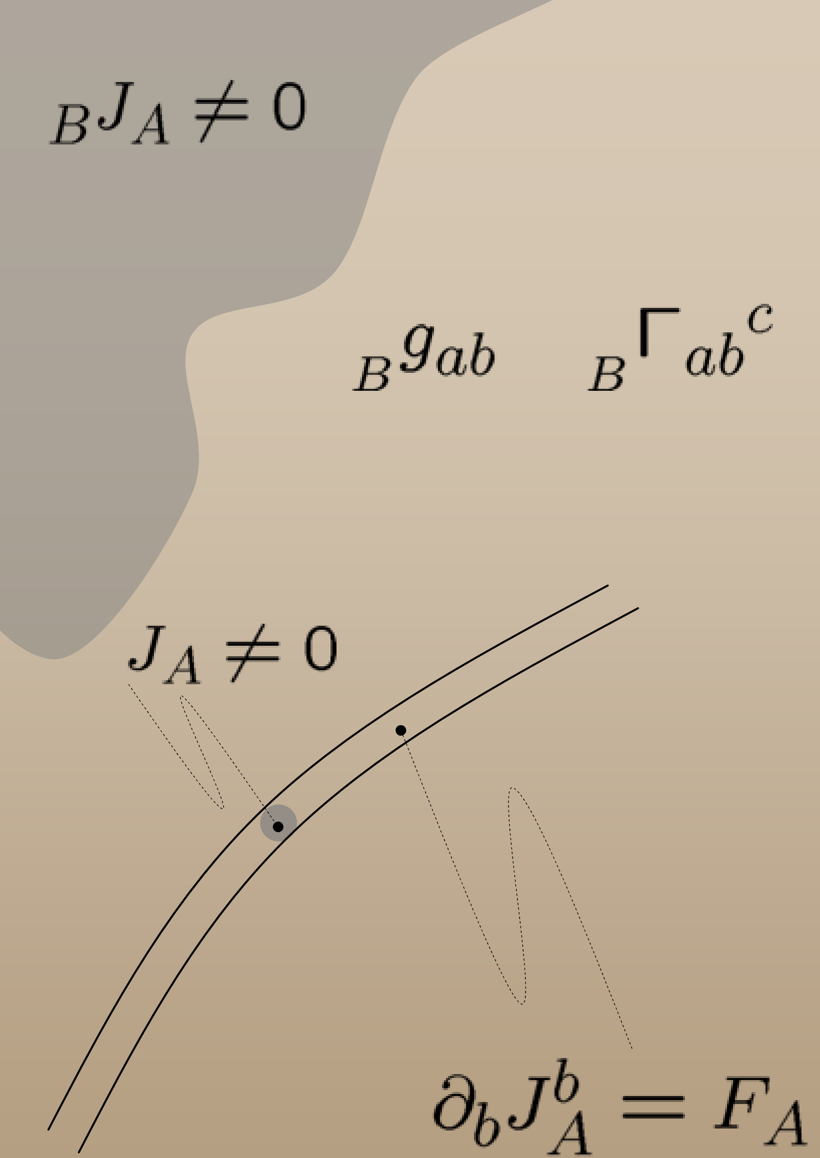
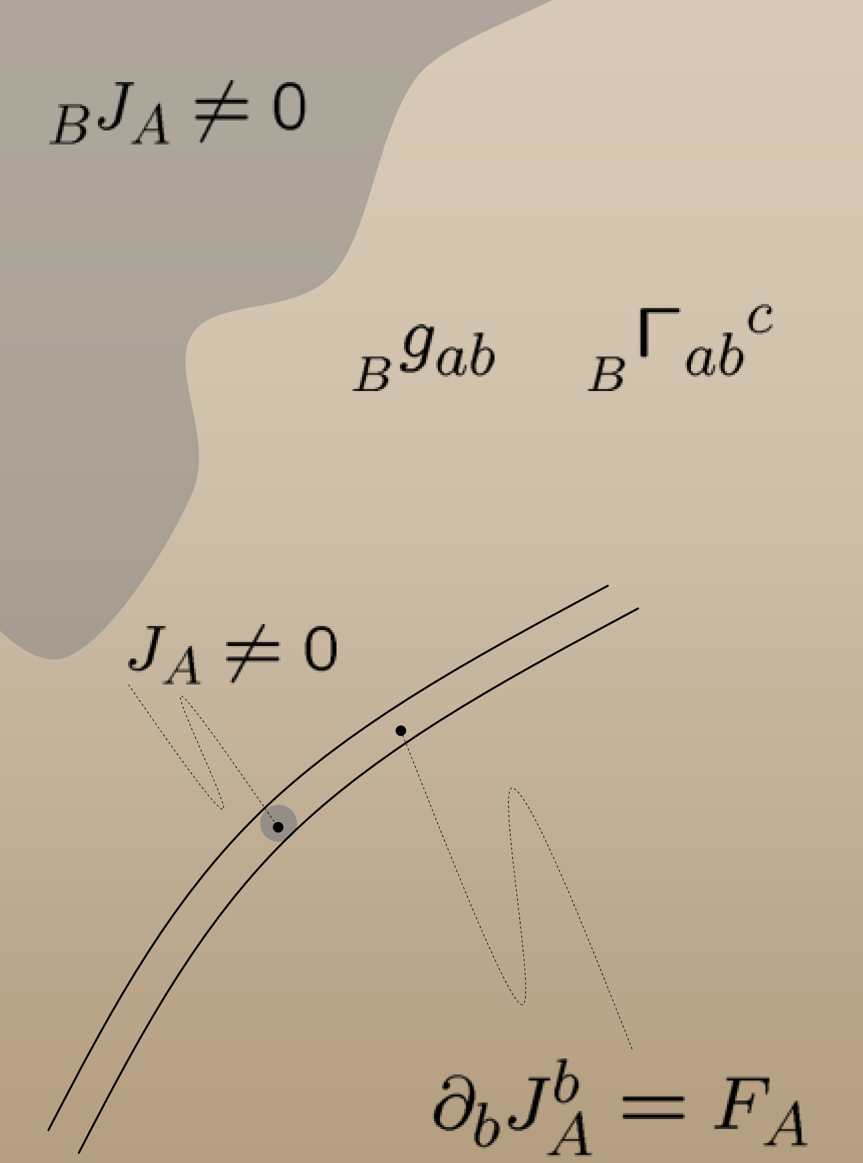
Joint Hannover-Golm Seminar
7 October 2008



${}_BJ_A \neq 0$
${}_Bg_{ab}$ ${}_B\Gamma_{ab}{}^c$
$J_A \neq 0$
$\partial_b J^b_A = F_A$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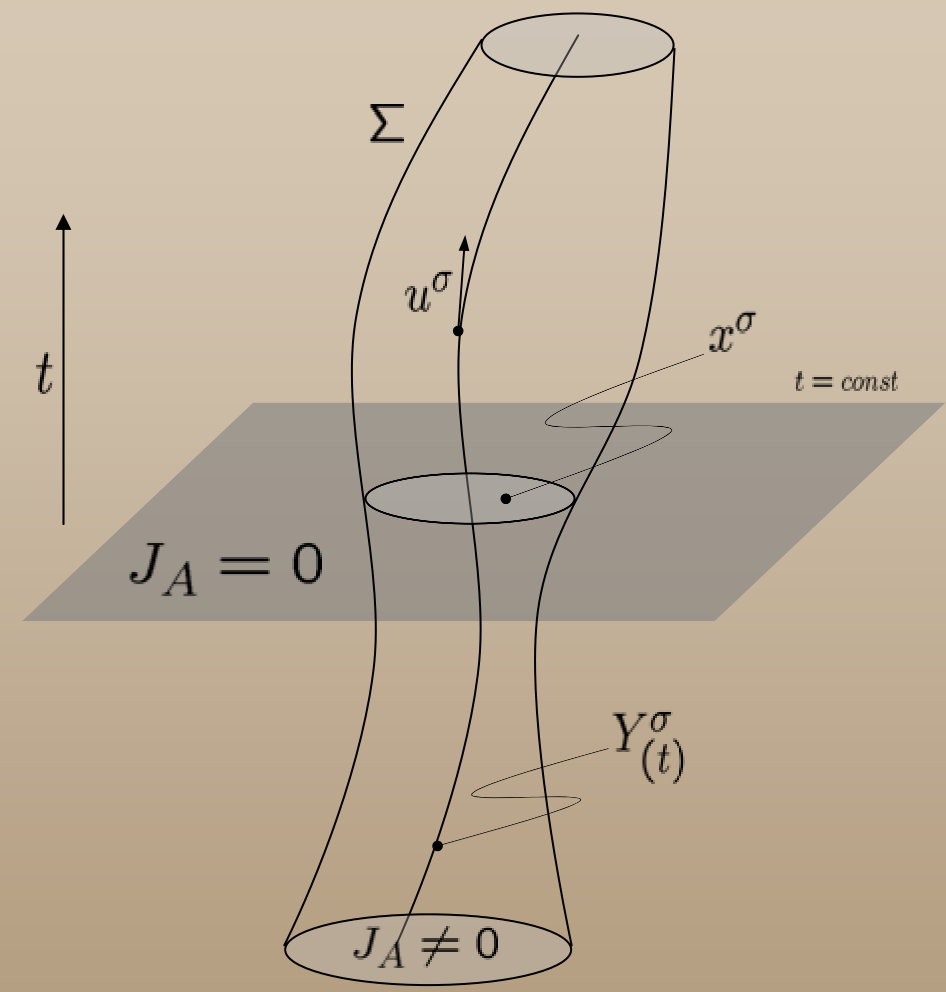
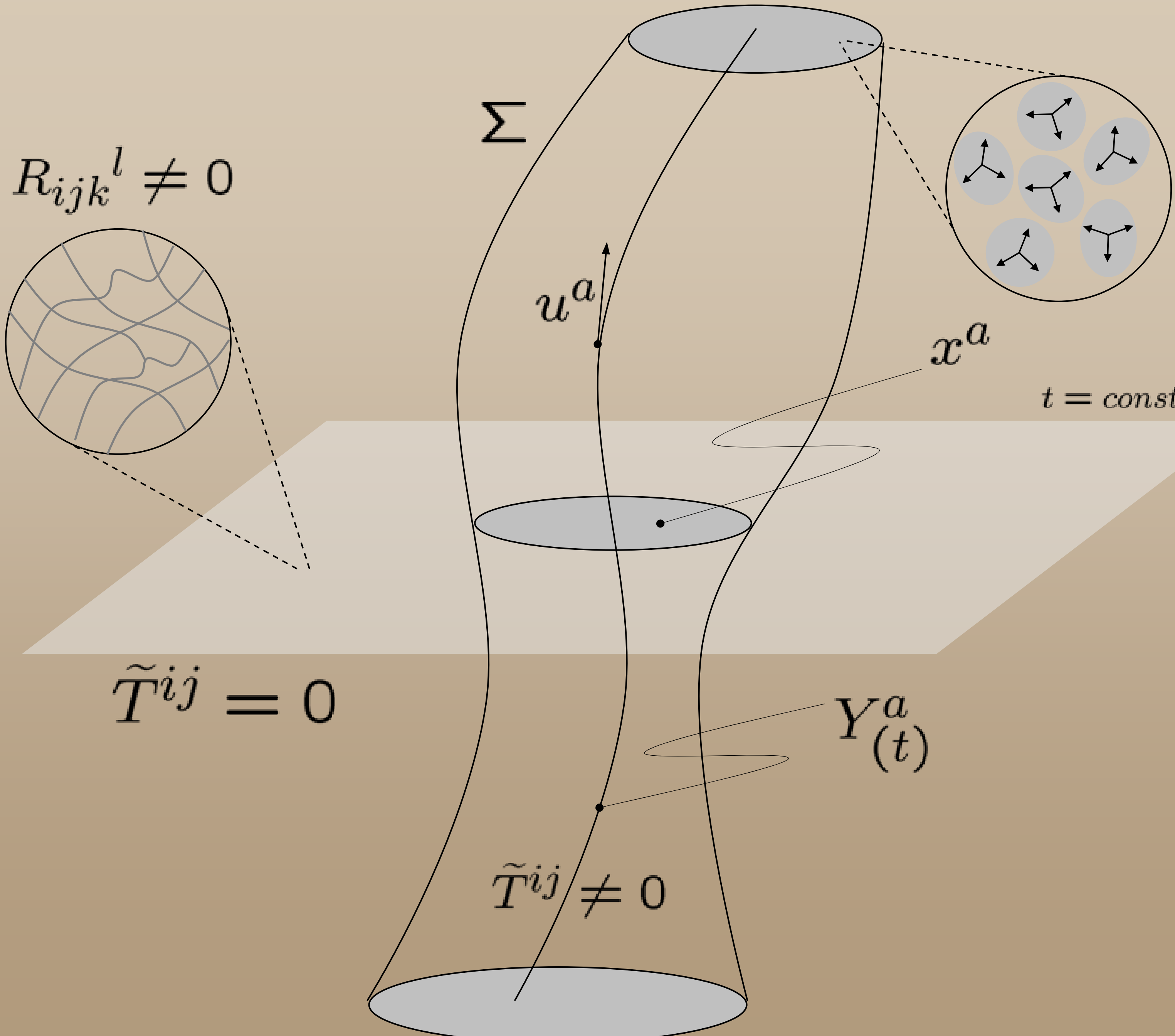

Σ
t
u^σ
x^σ
$t = const$
$J_A = 0$
$Y^\sigma_{(t)}$
$J_A \neq 0$



Σ
$R_{ijk}{}^{l} \neq 0$
u^a
x^a
$t = const$
$\widetilde{T}^{ij} = 0$
$Y^a_{(t)}$
$\widetilde{T}^{ij} \neq 0$

$$\frac{d}{dt}\int\left(\prod_{j=1}^{n}\delta x^{b_j}\right)J_A{}^0=\sum_{i=1}^{n}\rho^{b_i}{}_a\int\left(\prod_{j=1,\,j\neq i}^{n}\delta x^{b_j}\right)J_A{}^a+\int\left(\prod_{j=1}^{n}\delta x^{b_j}\right)J_A{}^a{}_{,a}$$

Integrated conservation law

$$\frac{d}{dt}\int\left(\prod_{j=1}^{n}\delta x^{b_j}\right)J_A{}^0=\sum_{i=1}^{n}\rho^{b_i}{}_a\int\left(\prod_{j=1,\,j\neq i}^{n}\delta x^{b_j}\right)J_A{}^a+\int\left(\prod_{j=1}^{n}\delta x^{b_j}\right)J_A{}^a{}_{,a}$$

Integrated conservation law

$$\begin{aligned}\delta x^a &= x^a - Y^a\\ \rho^b{}_a &= \delta x^b{}_{,a}=\delta^b_a - v^b\delta^0_a=\delta^b_a-\delta^b_0\delta^0_a=\delta^b_\alpha\delta^\alpha_a\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\int\left(\prod_{\alpha=1}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{i0}= &\sum_{\beta=1}^{n}\left[\int\left(\prod_{\alpha=1,\alpha\neq\beta}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{ib_\beta}-v^{b_\beta}\int\left(\prod_{\alpha=1,\alpha\neq\beta}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{i0}\right]\\ &-\int\left(\prod_{\alpha=1}^{n}\delta x^{b_\alpha}\right)\left(\overset{\{\}}{\Gamma}{}_{kj}{}^{i}\tilde{T}^{kj}\right)\end{aligned}$$

$$\frac{d}{dt}\int\left(\prod_{j=1}^{n}\delta x^{b_j}\right)J_A{}^0=\sum_{i=1}^{n}\rho^{b_i}{}_a\int\left(\prod_{j=1,\,j\neq i}^{n}\delta x^{b_j}\right)J_A{}^a+\int\left(\prod_{j=1}^{n}\delta x^{b_j}\right)J_A{}^a{}_{,a}$$

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$$\begin{aligned}\frac{d}{dt}\int\left(\prod_{\alpha=1}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{i0}= &\sum_{\beta=1}^{n}\left[\int\left(\prod_{\alpha=1,\alpha\neq\beta}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{ib_\beta}-v^{b_\beta}\int\left(\prod_{\alpha=1,\alpha\neq\beta}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{i0}\right]\\ &-\int\left(\prod_{\alpha=1}^{n}\delta x^{b_\alpha}\right)\left(\overset{\{\}}{\Gamma}{}_{kj}{}^{i}\tilde{T}^{kj}\right)\end{aligned}$$

$$\overline{T}^{b_1\cdots b_n ij} := \int\left(\prod_{\alpha=1}^{n}\delta x^{b_\alpha}\right)\tilde{T}^{ij}$$

$$\frac{d}{dt}\overline{T}^{b_1\cdots b_n i0}=\sum_{\beta=1}^{n}\left(\overline{T}^{b_1\cdots\breve{b}_\beta\cdots b_n ib_\beta}-v^{b_\beta}\overline{T}^{b_1\cdots\breve{b}_\beta\cdots b_n i0}\right)-\int\left(\prod_{\alpha=1}^{n}\delta x^{b_\alpha}\right)\left(\overset{\{\}}{\Gamma}{}_{kj}{}^{i}\tilde{T}^{kj}\right)$$

$$
\begin{aligned}
M^{ab} &:= u^0 \overline{T}^{ab} \\
M^{abc} &:= -u^0 \overline{T}^{abc} \\
M^{abcd} &:= -u^0 \overline{T}^{abcd} \\
S^{ab} &:= \overline{T}^{ab0} - \overline{T}^{ba0}
\end{aligned}
$$

$$
\frac{d}{ds}\left(\frac{M^{i0}}{u^0}\right) = -\overset{\{\}}{\Gamma}{}_{kj}{}^{i} M^{(kj)} + \overset{\{\}}{\Gamma}{}_{kj}{}^{i}{}_{,a} M^{a(kj)} + \frac{1}{2}\overset{\{\}}{\Gamma}{}_{kj}{}^{i}{}_{,ab} M^{ab(kj)} \qquad (1)
$$

$$
\begin{aligned}
&\frac{dS^{ai}}{ds} + \frac{u^a}{u^0}\frac{dS^{i0}}{ds} - \frac{u^i}{u^0}\frac{dS^{a0}}{ds} = -\left(\overset{\{\}}{\Gamma}{}_{kj}{}^{a} - \frac{u^a}{u^0}\overset{\{\}}{\Gamma}{}_{kj}{}^{0}\right) M^{i(kj)} + \left(\overset{\{\}}{\Gamma}{}_{kj}{}^{i} - \frac{u^i}{u^0}\overset{\{\}}{\Gamma}{}_{kj}{}^{0}\right) M^{a(kj)} \\
&-\left(\overset{\{\}}{\Gamma}{}_{kj}{}^{a}{}_{,b} - \frac{u^a}{u^0}\overset{\{\}}{\Gamma}{}_{kj}{}^{0}{}_{,b}\right) M^{ib(kj)} + \left(\overset{\{\}}{\Gamma}{}_{kj}{}^{i}{}_{,b} - \frac{u^i}{u^0}\overset{\{\}}{\Gamma}{}_{kj}{}^{0}{}_{,b}\right) M^{ab(kj)} \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&2M^{abi} + \left(u^b S^{ai} + u^i S^{ab}\right) - \frac{u^a}{u^0}\left(u^b S^{0i} + u^i S^{0b}\right) - \frac{u^a}{u^0}\overset{\{\}}{\Gamma}{}_{kj}{}^{0} M^{bi(kj)} \\
&-\overset{\{\}}{\Gamma}{}_{kj}{}^{i} M^{ab(kj)} + \overset{\{\}}{\Gamma}{}_{kj}{}^{a} M^{bi(kj)} - \overset{\{\}}{\Gamma}{}_{kj}{}^{b} M^{ia(kj)} = \\
&\frac{u^a}{u^0}\frac{d}{ds}\left(\frac{M^{bi00}}{u^0}\right) + \frac{d}{ds}\left[\frac{1}{u^0}\left(M^{abi0} - M^{bia0} + M^{iab0}\right)\right] \qquad (3)
\end{aligned}
$$

$$
2\left(M^{iabc} - M^{bcia}\right) = u^a M^{cib0} - u^b M^{aci0} - u^c M^{iba0} + u^i M^{bac0} \qquad (4)
$$

Quadrupolar equations of motion

Open questions

- Covariant form of the equations
- Supplementary condition(s)
- Relation to other approximation schemes
- (Numerical) integration
- Modelling of (real) sources