Effect of Higher Harmonics for Supermassive Black Hole Parameter Estimation

Ed Porter (AEI) & Neil Cornish (MSU) arXiv : 0804.0332

What is the effect of the inclusion of higher harmonic corrections (HHCs) on :

(a) luminosity distance, (b) sky resolution, (c) mass determination, (d) recovered SNR, (e) time to coalescence

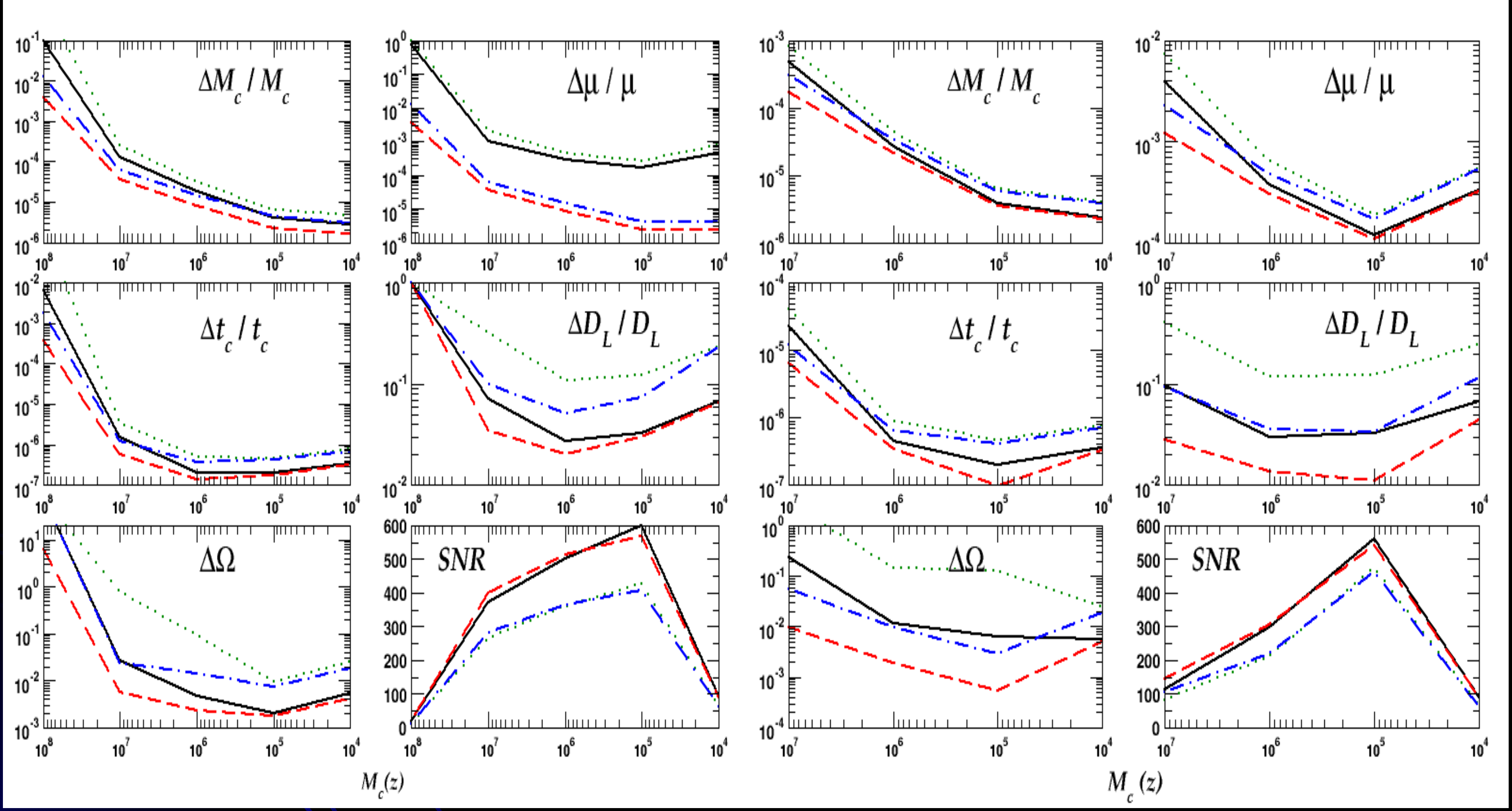
Also, If we lose detector links, is LISA dead or can we still conduct worthwhile astronomy?

The Monte Carlo Simulation

- 20,000 points
- 5 Different red-shifted chirp masses, Mass ratios of 1 and 10
- Standard luminosity distance of 10 Gpc
- Time of observation of 1 year, Time of coalescence of 0.999 yrs
- Low frequency approximation, Maximum allowed frequency of 5 mHz.
- All other parameters randomized.
- LISA cutoff frequency of 0.01 mHz with red noise.
- One detector (X) and two detector cases (AE)

Parameter Estimation

mr = 1 mr = 10



key : green dots (X RW), black solid (AE RW), Blue dash-dot (X HHCs), red dash (AE HHCs)

SMBH Inspiral Waveform and Error Estimation

The GW polarizations are given by

$$h_{+,\times} = \frac{2Gm\eta}{c^2 D_L}\left(\frac{Gm\omega}{c^3}\right)^{2/3}\left[H^{(0)}_{+,\times} + x^{1/2}H^{(1/2)}_{+,\times} + xH^{(1)}_{+,\times} + x^{3/2}H^{(3/2)}_{+,\times} + x^2 H^{(2)}_{+,\times}\right]$$

where $\vec{x} = \{\ln(M_c), \ln(\mu), \theta, \phi, \ln(t_c), \iota, \varphi_c, \ln(D_L), \psi\}$

and $x = \left(Gm\omega/c^3\right)^{2/3}$

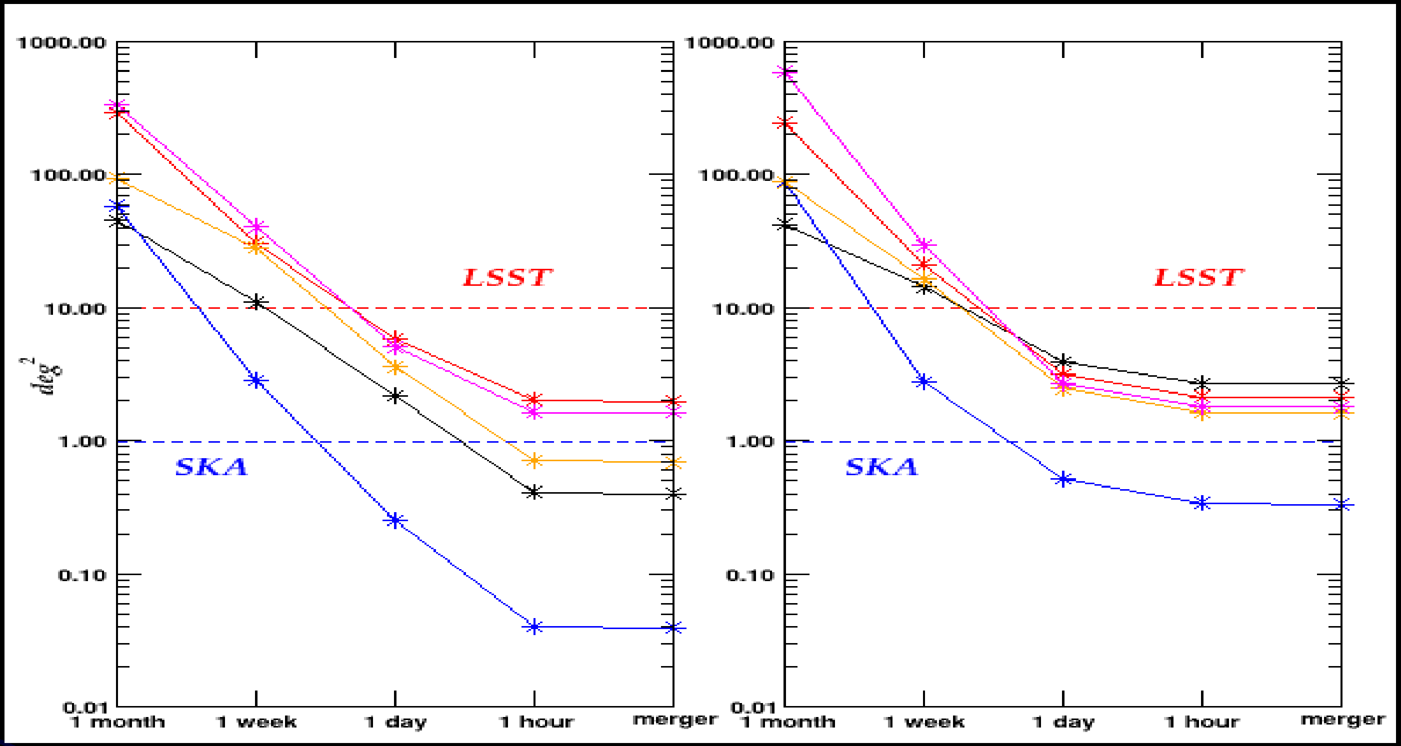
Using a Fisher matrix analysis, define $\Gamma_{\mu\nu} = \left\langle \frac{\partial h}{\partial \lambda^\mu} \middle| \frac{\partial h}{\partial \lambda^\nu} \right\rangle$

and the sky resolution error as $\Delta\Omega = 2\pi\sqrt{\Sigma^{\theta\theta}\Sigma^{\phi\phi} - \left(\Sigma^{\theta\phi}\right)^2}$

where $\Sigma^{ij} = \left\langle \Delta\lambda^i \Delta\lambda^j \right\rangle = \left(\Gamma^{ij}\right)^{-1}$

giving $\Sigma^{\theta\theta} = \langle \Delta\cos\theta \Delta\cos\theta \rangle$ and $\Sigma^{\phi\phi} = \langle \Delta\phi\Delta\phi \rangle$

Pre-Merger Localization for EM Counterpart Follow-ups



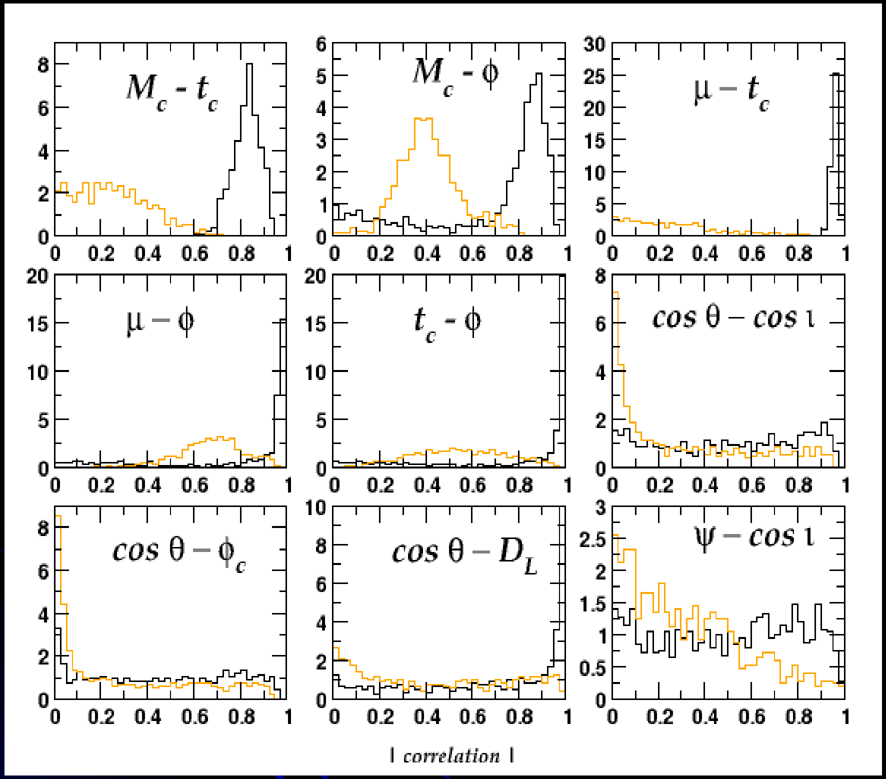
Conclusion

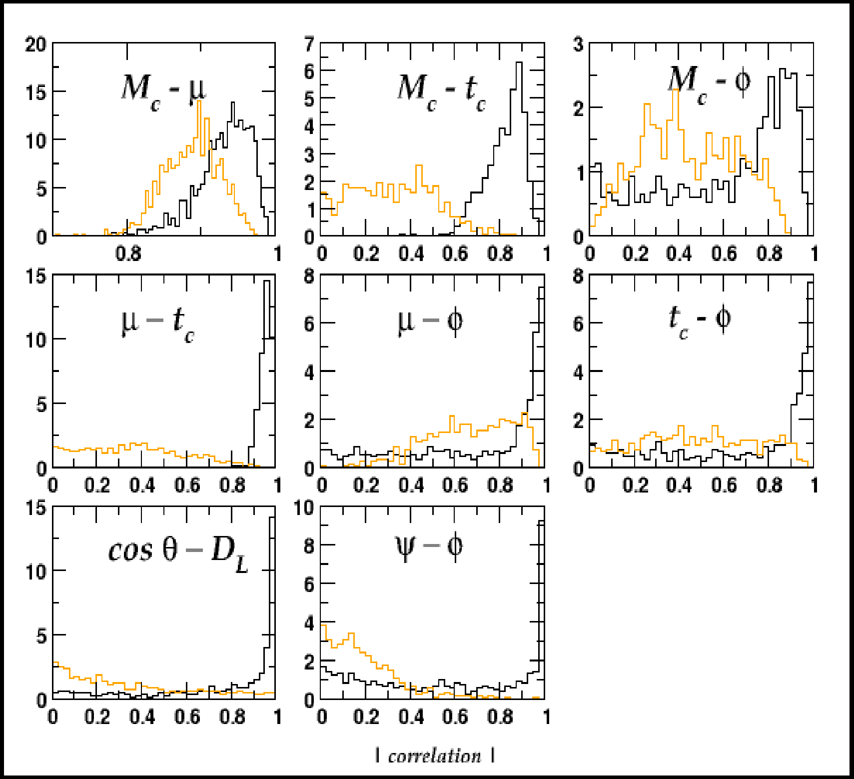
- HHCs allow better parameter estimation : more observables than unknowns.
- Can mitigate the loss of two detector links.
- Can allow higher mass systems to be visible even when dominant harmonic is not seen.
- Improves chances of EM follow-up.

Main Correlation Breaking

$$D^{ij} = \begin{cases} \sqrt{\Sigma^{ij}} & i = j \\ \Sigma^{ij} / \sqrt{\Sigma^{ii}\Sigma^{jj}} & i \neq j \end{cases}$$

mass ratio 10 : full and single detector LISA





Observables